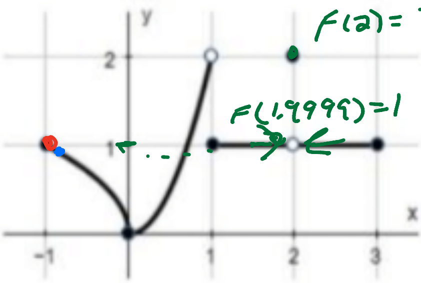
Period 1, 8/19/24

$\lim_{x\to 2} F(x)=1$



$F(2) = 2$

$F(1.9999)=1$

$F(2)=2$

Not continious

$\lim_{x\to 2} F(x) \neq F(2)$

$F(1.9999999)=1$

$F(2.0000000001)=1$

$> \lim_{x\to 2} F(x)=1$

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e) $\lim_{x\to c} f(x)$ exists at every c in the interval $(-1, 1)$

$-1<x<1$

$\lim_{x\to -.999} F(x) =$ close To 1

$[-1,1]$

$-1\le x\le 1$

$\rightarrow \lim_{x\to -1} F(x) = DNE$

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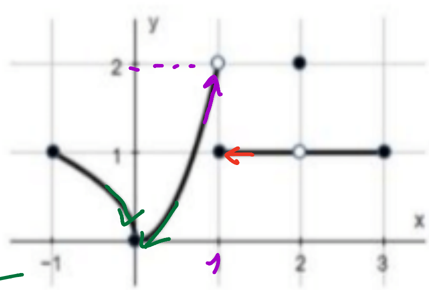
d) $\lim_{x\to 1^-} f(x) = 2$ True

left

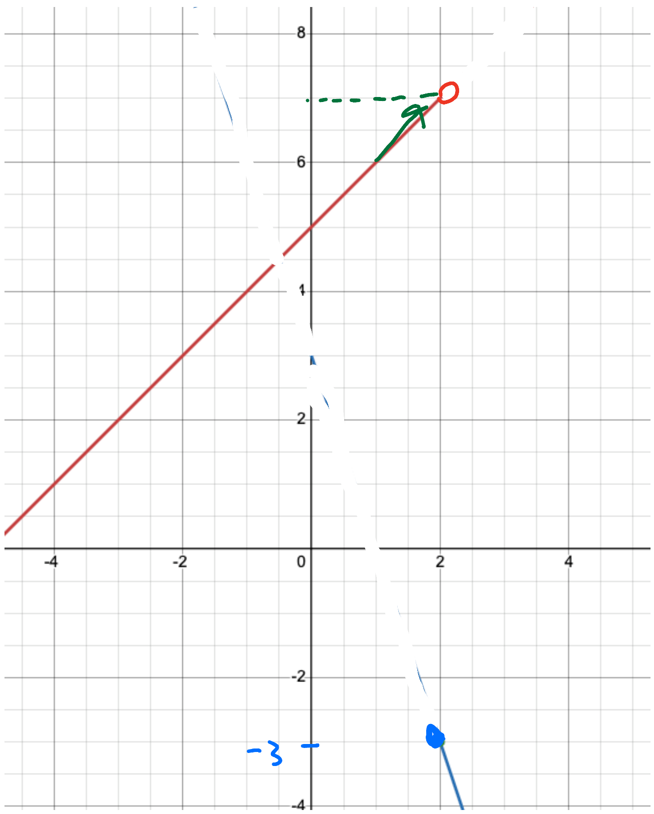


$\lim_{x\to 1^+} F(x)=1$

$\lim_{x\to 1} F(x) = DNE$

$\lim_{x\to 0^+} F(x) = \lim_{x\to 0^-} F(x)$

$0 = 0$



ıg: $f(x) = \begin{cases} x+5 & x<2 \\ -3 & x=2 \\ -3x+3 & x>2 \end{cases}$

$\lim\limits_{x\to 2^-} F(x) = 7$

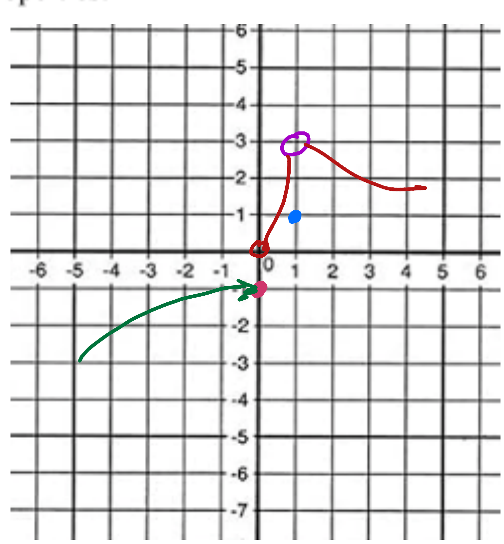
$\lim\limits_{x\to 2^+} F(x) = -3$

Not The Same

$\lim\limits_{x\to 2} F(X) = DNE = \varnothing$

4. Sketch a graph of a function with the given properties:

- ☐ $\lim\limits_{x\to 1} g(x) = 3$
- ☐ $\lim\limits_{x\to 0^-} g(x) = -1$
- ☐ $\lim\limits_{x\to 0^+} g(x) = 0$
- ☐ $g(0) = -1$
- ☐ $g(1) = 1$



i. $\lim_{x\to-\infty} f(x) = -1$

ii. $\lim_{x\to\infty} f(x) = 2$

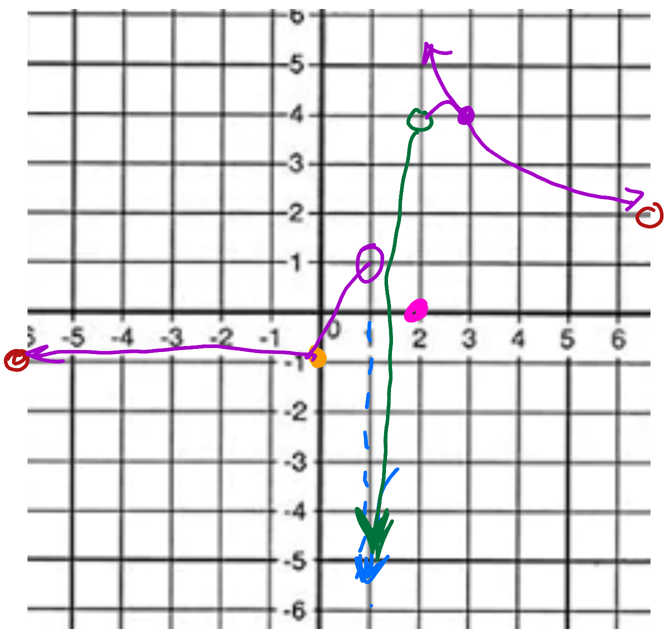
iii. $\lim_{x\to 1^+} f(x) = -\infty$

iv. $\lim_{x\to 2^-} f(x) = 4$

v. $f(2) = 0$

vi. $f(0) = -1$

vii. $f(3) = 4$



THEOREM SOME BASIC LIMITS		
Let b and c be real numbers and let n be a positive integer.		
1) $\lim_{x\to c} b = b$	2) $\lim_{x\to c} x = c$	3) $\lim_{x\to c} x^n = c^n$

1. $\lim_{x\to 4} 7 = 7$

$y = 7$

2. $y = x$

$\lim_{x\to 4} x = 4$

$\lim_{x\to -6} x = -6$

3. $\lim_{x\to 3} x^4 = 3^4 = 81$

$y = x^4$

THEOREM PROPERTIES OF LIMITS

Let b and c be real numbers and let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x\to c} f(x) = L \qquad \lim_{x\to c} g(x) = K$$

1. **Scalar multiple**: $\lim_{x\to c}[bf(x)] = bL$
2. **Sum or difference:** $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$
3. **Products:** $\lim_{x\to c}[f(x)g(x)] = LK$
4. **Quotient:** $\lim_{x\to c}\dfrac{f(x)}{g(x)} = \dfrac{L}{K}$, provided $K \neq 0$
5. **Power:** $\lim_{x\to c}[f(x)]^{n/m} = L^{n/m}$

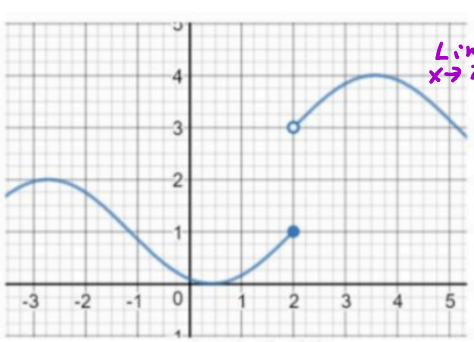
Optional Exra Practice

going to 2

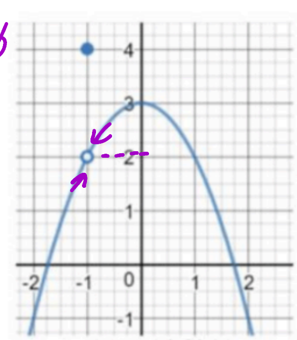
$\lim_{x\to -1} F(x) =$

1. The graphs of $g(x)$ and $f(x)$ are below. Find $\lim_{x\to -1} g(f(x))$.

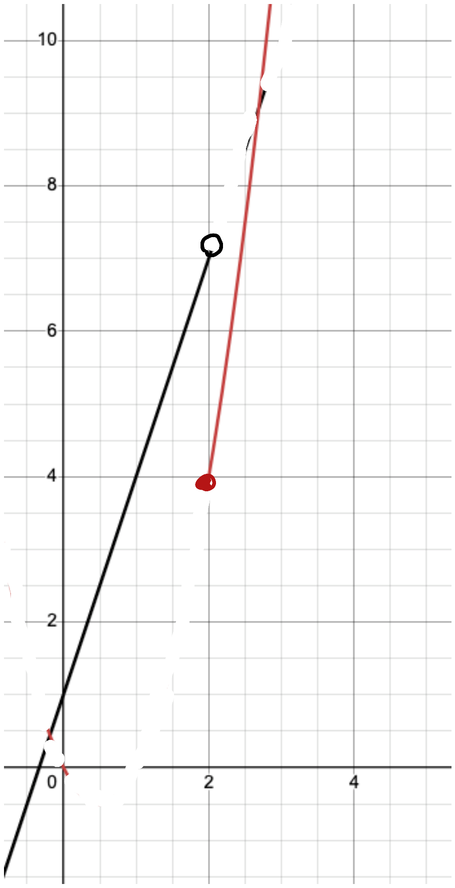
$\lim_{x\to 2} g(x) = \emptyset$



Graph of $g(x)$



Graph of $f(x)$



Find $\lim_{x\to 2} f(x)$, if it exist.

$$f(x) = \begin{cases} 3x+1 & if \quad x<2 \\ 2x(x-1) & if \quad x \geq 2 \end{cases}$$

$\lim_{x\to 2^-} F(x) = \lim_{x\to 2^-} (3x+1) = 7$

NoT Same

$\lim_{x\to 2^+} F(x) = \lim_{x\to 2} [2x(x-1)] = 4$

$\lim_{x\to 2} F(x) = \emptyset$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$x^3 - 1^3$

$$\lim_{x\to 1} \frac{x^3-1}{x-1}. = \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+1\cdot x+1^2)}{\cancel{(x-1)}} = 1^2+1\cdot 1+1^2 = 3$$

$\frac{0}{0} = \emptyset$

$$\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{x} = \frac{\frac{1}{1}-1}{0} = \frac{1-1}{0} = \frac{0}{0} = \emptyset = \lim_{x\to 0} \frac{-1}{x+1} = \frac{-1}{1} = -1$$

$$\frac{\frac{1}{x+1} - 1}{x} = \frac{\frac{1}{x+1} - \frac{x+1}{x+1}}{x} = \frac{\frac{\cancel{1}-x-\cancel{1}}{x+1}}{x} = \frac{\frac{-x}{x+1}}{\frac{x}{1}} = \frac{-\cancel{x}}{(x+1)} \cdot \frac{1}{\cancel{x}} = \frac{-1}{x+1}$$

$$\lim_{x \to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}. = \frac{\sqrt{0+1} \; -1}{0} = \frac{\sqrt{1}-1}{0} = \frac{1-1}{0} = \frac{0}{0}$$

$$= \emptyset$$

$$\frac{(\sqrt{x+1}-1)}{x} \cdot \frac{(\sqrt{x+1}+1)}{(\sqrt{x+1}+1)}$$

$$\frac{x+\cancel{1}+\cancel{\sqrt{x+1}}-\cancel{\sqrt{x+1}}-\cancel{1}}{x(\sqrt{x+1}+1)} = \boxed{\frac{\cancel{x}}{\cancel{x}(\sqrt{x+1}+1)}}$$

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}, = \text{Slope}$$

a) Find the average rate of change of $g(x) = x^2 + 3x$ from 2 to x, $x \neq 2$
b) Find the limit of part a as x goes to 2. $= 7$

$$\frac{g(x)-g(2)}{x-2} = \frac{x^2+3x-10}{x-2} = \frac{(x+5)\cancel{(x-2)}}{\cancel{x-2}}$$

The difference quotient of a function f at x is

$$\frac{f(x+h)-f(x)}{h}, \qquad h \neq 0$$

Example 11

a) For $t(x) = 2x^2 - 3x + 1$, find the difference quotient.

b) Find the limit of part a as h approaches 0.

$$\lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$$

$F(x+h) = 2(x+h)^2 - 3(x+h) + 1$

$F(x) = 2x^2 - 3x + 1$

$$\lim_{h\to 0} \frac{2(x^2+2xh+h^2) - 3(x+h) + 1 - [2x^2 - 3x + 1]}{h}$$

$$\lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-3x} - 3h \cancel{+1} \cancel{-2x^2} + \cancel{3x} \cancel{-1}}{h}$$

$$\lim_{h\to 0} \frac{4xh + 2h^2 - 3h}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} = \lim_{h\to 0} 4x + 2h - 3 =$$

$4x + 2(0) - 3$

$$\lim_{h\to 0} \frac{F(x+h) - F(x)}{h} = 4x - 3$$

12) $\lim_{x\to 0} \frac{\sin 4x}{x} = 4 \cdot 1 = 4$

$$\lim_{x\to 0} \frac{4}{4} \cdot \frac{\sin 4x}{x}$$

$$\lim_{x\to 0} 4 \cdot \frac{\sin 4x}{4x}$$

$$\lim_{\theta\to 0} \tan 6\theta \cdot \frac{1}{\theta}$$

13) $\lim_{\theta\to 0} \frac{\tan 6\theta}{\theta} =$

$$\frac{\sin 6\theta}{\cos 6\theta} \cdot \frac{1 \cdot 6}{\theta \cdot 6} = \frac{6 \sin 6\theta}{6\theta \cdot \cos 6\theta} = 1 \cdot \frac{6}{\cos 6\theta}$$

$$\lim_{\theta\to 0} \frac{6}{\cos 6\theta} = \frac{6}{1} = 6$$

$\cos \theta = 1$

$$\lim_{x\to 0}\frac{x^2}{1-\cos x} = \lim_{x\to 0}\frac{x^2(1+\cos x)}{\sin^2 x} = \lim_{x\to 0}\underbrace{\boxed{\frac{x^2}{\sin^2 x}}}_{=1}\cdot\frac{(1+\cos x)}{1} \Rightarrow$$

keep going

$$\sin^2 x+\cos^2 x=1$$

$$\frac{x^2}{(1-\cos x)}\cdot\frac{(1+\cos x)}{(1+\cos x)} = \frac{x^2(1+\cos x)}{1-\cancel{\cos x}+\cancel{\cos x}-\cos^2 x} = \frac{x^2(1+\cos x)}{1-\cos^2 x} = \frac{x^2(1+\cos x)}{\sin^2 x+\cancel{\cos^2 x}-\cancel{\cos^2 x}}$$

$$\sin(a+b) = \sin a\cos b+\sin b\cos a$$

Example 14

$$\lim_{x\to 0}\frac{\sin\left(\frac{\pi}{4}+x\right)-\sin\left(\frac{\pi}{4}\right)}{x} = \lim_{x\to 0}\frac{\sin\frac{\pi}{4}\cos x+\sin x\cos\frac{\pi}{4}-\sin\frac{\pi}{4}}{x}$$

$$\sin\frac{\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

$$=\lim_{x\to 0}\frac{\frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\sin x-\frac{\sqrt{2}}{2}}{x}$$

$$=\lim_{x\to 0}\frac{\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}}{x}+\frac{\frac{\sqrt{2}}{2}\sin x}{x}$$

$$\lim_{x\to 0}\frac{\sqrt{2}}{2}\underbrace{\boxed{\frac{(\cos x-1)}{x}}}_{=0}+\frac{\sqrt{2}}{2}\cdot\underbrace{\boxed{\frac{\sin x}{x}}}_{=1}$$

$$\lim_{x\to 0} \frac{\sin x}{5x^2 + x} = \lim_{x\to 0} \frac{\sin x}{x(5x+1)} = \lim_{x\to 0} \underbrace{\boxed{\frac{\sin x}{x}}}_{=1} \cdot \frac{1}{(5x+1)}$$

$$\lim_{x\to 0} x\csc(3x) = \lim_{x\to 0} \frac{x}{1} \cdot \frac{1}{\sin 3x} = \lim_{x\to 0} \frac{x}{\sin 3x} \cdot \frac{3}{3}$$

keep going